

Title of Metamaterials European School, September 1-2, 2017
Modelling of metamaterials:
numerical methods and homogenization techniques

Lecturers (by alphabetical order): Anne-Sophie Bonnet Bendhia (Paris, France), Elena Cherkav (Salt-Lake City, USA), Richard Craster (London, UK), Agnes Maurel (Paris, France), Ross McPhedran (Sydney, Australia), Sasha Movchan (Liverpool, UK), Alejandro Rodriguez (Princeton, USA), Mario Silveirinha (University of Lisbon), Igor Tsukerman (Akron, USA) and Martin Wegener (Karlsruhe, Germany),

Preliminary Program- Day 1 (Friday September 1st)

8:00-8:50 Registration

8:50-9:00 Opening Ceremony

9:00-10:20: Richard Craster (Imperial College London, UK): Elastic metamaterials and high frequency homogenisation

The abstract:

This will be two slightly disjoint presentations. First I will present ideas around creating metasurfaces in elasticity using subwavelength structuration. It has been known for many years now that one can develop metamaterials in electromagnetism and, at least in theory, steer and manipulate waves in optics and electromagnetism; although in reality the various losses in real materials do limit the effectiveness. This is all at the nanoscale, elasticity and acoustics, on the other hand, are typically at far longer scales and the effect of losses can be less. I will consider two different models: one based on subwavelength resonators to generate mode conversion from surface to bulk waves which is truly a metamaterial and then I will introduce a seismic metamaterial based on periodic media with an ultra-low and wide bandgap that will stop very long waves from propagating, so subwavelength in some sense but not using resonators.

Second, I will return purely to theory and explore how we can make sense of apparently complicated dispersion diagrams by showing that one can homogenise in regimes that are far from the conventional long-wave regime. The talk will introduce high frequency homogenisation and show how these ideas can be used to interpret and design photonic, phononic crystals and mechanical and seismic metamaterials.

10:20-10:40 Coffee Break

10:40-12:00: Agnes Maurel (ESPCI Paris & CNRS, France): Homogenization of thin interfaces

The abstract:

A homogenization approach will be presented to derive effective jump conditions applying across a microstructured interface. The homogenization is based on two scale expansions of the field in the near and far fields and on the asymptotic matching of the near and far fields. This allows to build an equivalent problem where the effect of the evanescent field is encapsulated in effective parameters entering in jump conditions. The validity of the equivalent problem will be inspected in several scattering problems.

This lecture is based on the joint work with J.-J. Marigo.

12:00-13:20 Lunch

13:20-14:40: Ross McPhedran (Sydney University, Australia): Multipole methods in photonics and platronics

The abstract:

In this review, I will discuss multipole methods, problems for which they are well adapted, and some of their advantages over competing methods.

I will commence with their applications to the study of composite materials, where multipole expansions are used in deriving for periodic geometries effective media formulae which take into account higher order poles. I will go on to their application to the Helmholtz equation, and to photonic crystals. A particularly successful application I will then discuss is to the theory of photonic crystal fibres, where multipole methods were the first to elucidate symmetry properties of modes, and their confinement losses in finite structures.

I will then go on to the topic of multipole methods applied to cloaking. This will be both for the Helmholtz equation (photonics) and for the biharmonic equation (platronics).

Although the main discussion will focus on doubly-periodic systems in two dimensions, I will discuss (if time permits) work on arrays of spheres and recent results on scattering properties of high-index dielectric spheres.

14:40-15:00 Coffee Break

15:00-16:20: Alexander Movchan (University of Liverpool, UK): Chiral elastic metamaterials and directional localisation

The abstract:

Geometrically chiral periodic media are straightforward to design and construct. However, these are very different from true chiral periodic systems which support directional preference in propagation of elastic waves. The lecture is based on several recent publications addressing the properties of elastic Floquet-Bloch waves in lattices with built-in gyroscopes. These create efficient filters and polarisers of elastic waves. In particular, one can design a multi-scale chiral elastic medium, which supports only shear waves. Dispersion properties of Floquet-Bloch waves are accompanied by the analysis of defect modes as well as spatial localisation. One of counter-intuitive and striking illustrations includes a system of gyros embedded into an elastic periodic lattice in such a way that it supports a localised waveform along a rhombus.

This lecture is based on the joint work with I.S. Jones, M. Brun, G. Carta, N. Movchan and M. Nieves.

16:20-17:40: Anne-Sophie Bonnet Bendhia (ENSTA Paris & CNRS, France): Numerical models for sign-shifting permittivity media

The abstract:

In almost all interesting configurations involving metamaterials or plasmonic devices, there is an interface between a classical dielectric material and a material with a negative dielectric permittivity and/or a negative magnetic permeability. If their imaginary parts are small (which one wishes for the applications), this may lead to difficulties when finite element computations are performed. In particular, when the interface present corners or edges, the convergence may fail.

During this lecture, I will explain that this is due to an unusual singular behavior of the electromagnetic field, which takes for some configurations the features of a so-called "black-hole" wave, whose energy

is trapped by the corners. During this lecture, I propose to describe these phenomena and to give the receipts, which ensure the convergence of the numerical method.

In configurations without blackhole effect, we show that the expected convergence is ensured as soon as the mesh satisfies some precise rules at the corners. But in presence of blackhole effect, this is not sufficient. The solution that we have found and validated is based on an original use of Perfectly Matched Layers at the corners.

17:40 – 18:00: Q&A Panel Session

18:00-21:00: Social event: Dinner party on boat

Preliminary Program- Day 2 (Saturday September 2nd)

9:00-10:20: Elena Cherkaev (University of Utah, USA): Inverse homogenization: An inverse problem for the structure of composites

The abstract: The resolvent representation of the effective properties of an anisotropic composite material results in a Stieltjes integral representation with a matrix-valued measure. I will discuss applications of this integral representation to forward and inverse homogenization. Inverse homogenization is a problem of deriving information about the microgeometry of a finely structured material from its known effective properties. The approach is based on the reconstruction of the matrix-valued spectral measure in the Stieltjes representation of the effective properties. This representation relates the n -point correlation functions of the microstructure to the moments of the spectral measure of an operator depending on the composite's geometry. The spectral measure contains all information about the structure of the material and can be uniquely recovered from effective measurements known in an interval of frequency. I will discuss identification of microstructural parameters from electromagnetic and viscoelastic effective measurements and show an extension of the approach to nonlinear materials. The resulting spectroscopic imaging method uses Pade approximants that provide an efficient way to construct spectrally matched microstructures.

10:20-10:40: Coffee Break

10:40-12:00: Martin Wegener (Karlsruhe Institute for Technology, Germany): 3D metamaterials: It all starts with the design

The abstract:

Designing metamaterials is a bit like arts. You have to be creative. Come up with new ideas for unusual effective material behavior that possibly nobody has ever seen before. Then comes the sweat. To test the idea under realistic assumptions, numerical calculations are quite often a must. Structures need to be optimized iteratively given certain constraints. Gaining an intuition is crucial. On the basis of the gained blueprints, we then fabricate 3D metamaterials by 3D laser nanoprinting. Finally, measurements on these samples are compared with numerical results to assess the quality of the achieved.

In this lecture, I will discuss selected recent examples for the above process. This includes mechanical metamaterials (pentamode, buckling, negative thermal expansion, negative static compressibility, micropolar, micromorphic) and semiconductor metamaterials (sign-reversed Hall-coefficient, parallel Hall effect).

12:00-13:20 Lunch

13:20-14:40: Mario Silveirinha (University of Lisbon, Portugal): Bridging Electromagnetic and Quantum Metamaterials: A unified homogenization formalism

The abstract: I will present an overview of our work on the characterization of electromagnetic and quantum metamaterials using effective medium methods [1-5]. It will be highlighted that both electromagnetic and electronic systems can be homogenized in a unified manner based on the introduction of an effective Hamiltonian operator that describes the time evolution of the macroscopic initial states as well as the stationary states of the relevant system. Particular emphasis will be given to (i) wire metamaterials in the case of electromagnetic waves and to (ii) artificial graphene platforms in the case of matter waves. It will be shown that complex metallic networks (wire media) can be modeled very precisely based on a nonlocal effective response and additional boundary conditions. Moreover, it will be shown that the time evolution of electron waves in highly anisotropic graphene superlattices can be described with a “mean-field” Hamiltonian that regards the material as a continuum.

[1] M. G. Silveirinha, A Metamaterial Homogenization Approach with Application to the Characterization of Microstructured Composites with Negative Parameters, *Phys. Rev. B*, 75, 115104, 2007 & Time Domain Homogenization of Metamaterials, *Phys. Rev. B*, 83, 165104, 2011.

[2] M. G. Silveirinha, N. Engheta, Effective Medium Approach to Electron Waves: Graphene Superlattices, *Phys. Rev. B*, 85, 195413, 2012.

[3] M. G. Silveirinha, N. Engheta Transformation Electronics: Tailoring the Effective Mass of Electrons, *Phys. Rev. B*, (Rapid Communication), 86, 161104(R), 2012.

[4] S. Lannebère, M. G. Silveirinha, Effective Hamiltonian for electron waves in artificial graphene: A first principles derivation, *Phys. Rev. B*, 91, 045416, 2015.

[5] D. E. Fernandes, M. Rodrigues, G. Falcão and M. G. Silveirinha, Time Evolution of Electron Waves in Graphene Superlattices, *AIP Advances*, 6, 075109, 2016.

14:40-15:00 Coffee Break

15:00-16:20: Alejandro Rodriguez (University of Princeton, USA): Computational methods and large-scale inverse design in nanophotonics: from nonlinear optics to thermal radiation

The abstract:

As the power requirements of devices continues to scale down, nonlinear optical interactions and even tiny effects due to thermal and quantum fluctuations of matter can have a profound impact on their functionalities. We survey recent computational methods and large-scale inverse-design techniques which have begun to shed light on decades-old as well as emerging problems in nanophotonics. From nonlinear frequency conversion and single-photon emission to emerging problems in the areas of thermal radiation and radiative heat transport, a commonality among these is the need to confine and tailor light over broad bandwidths and small volumes (with applications to thermal photovoltaics and single-photon sources), creating challenges for traditional photonic design methodologies based on index guiding or band-gap engineering.

16:20-17:40: Igor Tsukerman (University of Akron, USA): Effective medium theories backward in time: From the 21st to the 19th century (Non-Asymptotic and Nonlocal Approximations, Finite Samples, Interface Boundaries)

The abstract:

Research in electromagnetic metamaterials started three decades ago, if not earlier, and exploded in the 2000s as a quest for “perfect lenses,” “perfect absorbers,” etc. But, as the field of metamaterials matured, it became clear that ideal devices were not realizable because of losses, finite lattice cell sizes, and other factors. Undoubtedly, however, “imperfect” materials and devices will continue to be developed, and

we can therefore expect a growing need for more sophisticated methods of their analysis and, more specifically, for accurate homogenization theories valid for any composition and size of the lattice cell.

The objective of homogenization (effective medium theory) is to describe a composite structure in terms of effective parameters accurately representing reflection, transmission and propagation of waves on the scale coarser than the lattice cell size.

The lecture introduces a homogenization methodology valid in both electrostatics and electrodynamics and applicable to an arbitrary size and composition of the lattice cell. Nonlocal effects can be included in the model, making order-of-magnitude accuracy improvements possible.

We then travel backward in time and explore the connection between the new framework and the classical 19th – early 20th century theories of Clausius-Mossotti, Lorenz-Lorentz, Maxwell Garnett.

This lecture is based on the joint work with V. Markel.

17:40 – 18:00: Closing remarks and farewell